

## Consideration on Turbulent Transport of Combined Convection Using Modified Two-equation Turbulence Models

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### ABSTRACT

Recently, the two-equation turbulence models for simulating velocity fields have been improved variously. It becomes possible that the turbulence models accurately estimate turbulent statistic such as Reynolds stress near wall and the wall asymptotic behavior. The two-equation turbulence models have also been developed on temperature fields, and the models are useful for simulating another turbulent statistic such as turbulent heat fluxes near wall. However, the turbulent combined convection strongly affected by buoyancy, there is little research effort in which an appropriate turbulence model is made to reflect the buoyant effect accurately. In general, it is often difficult to measure the turbulent statistic of the combined convection at a good accuracy, because the influence of temperature fluctuation on various sensors is rather large. Analyzing the turbulent transport phenomena strongly affected by the buoyancy has not even been discussed empirically owing to the difficulty of the measurement. In particular, the behavior of the turbulent Prandtl number has not been addressed numerically and empirically for the buoyant dominant flow such as the combined convection.

Therefore, in this study, the buoyant components were introduced to the turbulent fluxes such as Reynolds stress and turbulent heat fluxes; the turbulent transport phenomena of the combined convection were modeled by adding several buoyant-induced components to the expressions of the turbulent fluxes. The turbulent transport mechanism of the combined convection was also examined by applying a new damping function being applicable to both the aiding and the opposing flows. The calculation target is the fully-developed turbulent combined convection between vertical smooth parallel plates, where the inertia force acts on parallel or opposite to the buoyant force. The wall is heated uniformly. The numerical computation was performed by combining the two-equation turbulence models of the different type being applicable to both the velocity and the temperature fields. It was revealed from a series of computations that the utilized model simulates the heat transfer and fluid flow of the turbulent combined convection physically.

Confirming the applicability of the two-equation turbulence models to the turbulent combined convection is important. Realizing the industrial convenience by using more appropriate and simpler turbulence models with the aid of the empirical knowledge is also necessary.

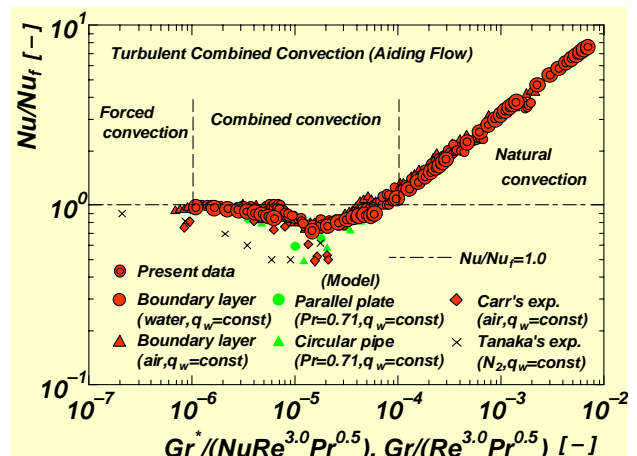


Figure A-1 Heat Transfer Coefficients of Turbulent Combined Convection (Aiding Flow)

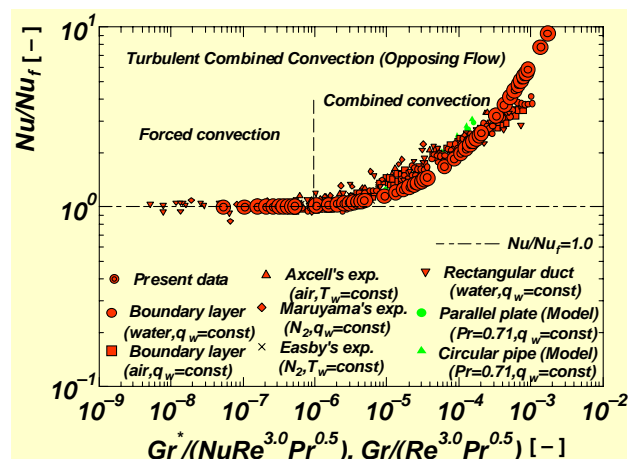


Figure A-2 Heat Transfer Coefficients of Turbulent Combined Convection (Opposing Flow)

## INTRODUCTION

The gravity works in all forced convection fields with heat transfer on the earth. The buoyancy is inevitably arising owing to the local temperature gradient. Therefore, in a wide sense, the flow is a convective heat transfer problem with combined natural convection and forced convection. We can say that the forced convection is handled as one limit that the buoyancy is negligible. On the contrary, in natural convection under a limited condition that is combined with forced convection, there is the other limit that the buoyancy becomes more dominative than the inertia force. We can say that the natural convection is the other limit that the inertia force is negligible. In a narrow sense, various heat transfer correlations are useful in both natural convection and forced convection under the limited conditions whether the buoyancy plays a significant role in heat transfer or not. Therefore, it is very important to grasp in what kind of case the conditions are satisfied industrially. The relative effect on the flow direction in which the buoyant force and the inertial force work should also be clarified.

Recently, the two-equation turbulence models for simulating velocity fields have been improved variously [for example, 1, 2, 3, 4]. It becomes possible that the turbulence models accurately estimate turbulent statistic such as Reynolds stress near wall and the wall asymptotic behavior. The two-equation turbulence models have also been developed on temperature fields [for example, 5], and the models are useful for simulating another turbulent statistic such as turbulent heat fluxes near wall. However, these turbulence models have mainly been developed for simulating the turbulent phenomenon of forced convection. For the turbulent combined convection strongly affected by the buoyancy, there is little research effort in which an appropriate turbulence model is made to reflect the buoyant effect accurately. In general, it is often difficult to measure the turbulent statistic of the combined convection at a good accuracy. This is because the measured data are accompanied by a large uncertainty due to the large temperature gradient and the variation that originate from the buoyancy, even if the latest LDV, PTV or PIV methods are adopted. Analyzing the turbulent transport phenomena strongly affected by the buoyancy has not even been discussed empirically owing to the difficulty of the measurement. In particular, the behavior of the turbulent Prandtl number has not been addressed numerically and empirically for the buoyant dominant flows such as the combined convection. As well as natural convection, not only the numerical data but also the empirical data of the turbulent Prandtl number distribution have not been clarified either for both natural convection and combined convection under the strong buoyant control. There are overwhelmingly small experimental databases in which the reliance to be used for a modeling is possible. The numerical modeling for the combined convection had to be handled in a way similar to the modeling of forced convection.

Therefore, in this study, the buoyant components were

introduced to the turbulent fluxes such as Reynolds stress and turbulent heat fluxes; the turbulent transport phenomena of the combined convection were modeled by adding several buoyant-induced components to the expressions of the turbulent fluxes in accordance with the turbulence model [6]. Furthermore, the turbulent transport mechanism of the combined convection was also examined by applying a new damping function being applicable to both the aiding and the opposing flows. The damping function was composed of a gradient of square root of turbulent kinetic energy near wall instead of using a friction velocity. The calculation target is the fully-developed turbulent combined convection between vertical smooth parallel plates, where the inertia force acts on parallel or opposite to the buoyant force. The wall is heated uniformly. The flow is one of the most fundamental systems in which the buoyancy markedly works. The Prandtl number was made to be 0.72 of air. The numerical computation was performed by combining the two-equation turbulence models of the different type being applicable to both the velocity and the temperature fields. From an industrial point of view, it is very important that the turbulent transport mechanism should be clarified as much as possible by using appropriate turbulence models when the turbulent statistic is difficult to measure quantitatively.

## NOMENCLATURE

- $C_f$  = friction coefficient [-]
- $C_b, C_h, C_1, C_2, C_\mu$  = turbulence model constants for velocity field [-]
- $C_{d1}, C_{d2}, C_{p1}, C_{p2}, C$  = turbulence model constants for temperature field [-]
- $C_p$  = specific heat at constant pressure [J/kgK]
- $f_u, f_1, f_2$  = turbulence model functions for velocity field [-]
- $f, f_{p1}, f_{p2}, f_{d1}, f_{d2}$  = turbulence model functions for temperature field [-]
- $Gr$  = Grashof number  $g \Delta TH^3 / \nu^2$  [-]
- $Gr^*$  = Modified Grashof number  $g q_w H^4 / \nu^2$  [-]
- $G$  = gravitational acceleration [m/s<sup>2</sup>]
- $H$  = width between vertical parallel plates, hydraulic diameter [m]
- $Nu$  = Nusselt number [-]
- $P$  = pressure in momentum equation [Pa]
- $Pr$  = molecular Prandtl number of fluid / [-]
- $Pr_t$  = turbulent Prandtl number [-]
- $q_w$  = wall heat flux [W/m<sup>2</sup>]
- $Re$  = Reynolds number  $U_m H / \nu$  [-]
- $R_t$  = turbulent Reynolds number  $\nu / \lambda$  [-]
- $T$  = mean temperature [K]
- $T_c$  = temperature at center [K]
- $T_w$  = wall temperature [K]
- $T_m$  = bulk mean temperature [K]
- $t$  = fluctuating temperature [K]
- $t^2$  = intensity of fluctuating temperature [K<sup>2</sup>]
- $U_m$  = bulk mean velocity [m/s]
- $U_i$  = vectors of mean velocity [m/s]
- $u_c$  = fluctuating velocity based on combined convection [m/s]
- $u_f$  = fluctuating velocity based on forced convection [m/s]
- $u_i$  = vectors of fluctuating velocity [m/s]
- $u_n$  = fluctuating velocity based on natural convection [m/s]

$u^*$  = friction velocity [m/s]  
 $u^*$  = friction velocity based on square root of turbulent kinetic energy near wall [m/s]  
 $uv$  = Reynolds stress [ $m^2/s^2$ ]  
 $u_i, t$  = vectors of turbulent heat flux [mK/s]  
 $Var_i$  = variables of transport equations [-]  
 $x$  = coordinate parallel to flow [m]  
 $y$  = coordinate perpendicular to wall [m]  
 $y^+$  = non-dimensional coordinate perpendicular to wall  $yu^*/\nu$  [-]  
 $y^*$  = non-dimensional coordinate perpendicular to wall  $yu^*/\nu$  [-]

#### Greek symbols

$\nu$  = molecular thermal eddy diffusivity of fluid [ $m^2/s$ ]  
 $\nu_t$  = turbulent thermal eddy diffusivity [ $m^2/s$ ]  
 $\beta$  = coefficient of volume expansion of fluid [1/K]  
 $\epsilon$  = dissipation rate of turbulent kinetic energy [ $m^2/s^3$ ]  
 $\epsilon_t$  = dissipation rate of fluctuating temperature [ $K^2/s$ ]  
 $k$  = turbulent kinetic energy [ $m^2/s^2$ ]  
 $k_f$  = heat conductivity of fluid [W/mK]  
 $\mu$  = molecular eddy viscosity of fluid [ $m^2/s$ ]  
 $\mu_t$  = turbulent eddy viscosity [ $m^2/s$ ]  
 $\rho$  = density of fluid [ $kg/m^3$ ]  
 $C_{\mu}$  = turbulence model constants for velocity field [-]  
 $C_{\mu_t}$  = turbulence model constants for temperature field [-]  
 $t$  = time [s]  
 $t_m$  = mixing time scale [s]

#### Superscripts

$+$  = non-dimensional coordinate  
 $*$  = near wall, modified

#### Subscripts

$c$  = based on combined convection, center  
 $f$  = based on forced convection  
 $i$  = vectors  
 $m$  = based on bulk mean value  
 $n$  = based on natural convection  
 $t$  = turbulent  
 $w$  = at wall

## MODELING AND NUMERICAL PROCEDURE

### Transport Equation

The present transport equations were made to couple the two-equation turbulence models for both the velocity and the temperature fields that follow the proposal [6]. They are as follows under the fully-developed conditions of the turbulent combined convection between vertical smooth parallel plates, where the inertia force acts on parallel or opposite to the buoyant force,

#### Momentum transport

$$0 = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\partial}{\partial y} \left( \nu \frac{\partial U}{\partial y} - uv \right) \pm g\beta(T - T_m)$$

#### Energy transport

$$U \frac{\partial T_m}{\partial x} = \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} - vt \right)$$

#### Turbulent kinetic energy transport

$$0 = \frac{\partial}{\partial y} \left( \left( \nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial y} \right) - uv \frac{\partial U}{\partial y} \pm g\beta ut - \epsilon - 2\nu \left( \frac{\partial \sqrt{k}}{\partial y} \right)^2$$

#### Dissipation rate of turbulent kinetic energy

$$0 = \frac{\partial}{\partial y} \left( \left( \nu + \frac{\nu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial y} \right) + \frac{\epsilon}{k} \left\{ C_{\epsilon 1} f_1 \left( -uv \frac{\partial U}{\partial y} \pm g\beta ut \right) - C_{\epsilon 2} f_2 \epsilon \right\} + \nu \nu_t \left( 1 - f_\mu \right) \left( \frac{\partial^2 U}{\partial y^2} \right)^2$$

#### Fluctuating temperature transport

$$0 = \frac{\partial}{\partial y} \left( \left( \alpha + \frac{\alpha_t}{\sigma_h} \right) \frac{\partial t^2}{\partial y} \right) - 2ut \frac{\partial T}{\partial x} - 2vt \frac{\partial T}{\partial y} - 2\epsilon_t - 2\alpha \left( \frac{\partial \sqrt{t^2}}{\partial y} \right)^2$$

#### Dissipation rate of fluctuating temperature

$$0 = \frac{\partial}{\partial y} \left( \left( \alpha + \frac{\alpha_t}{\sigma_\phi} \right) \frac{\partial \epsilon_t}{\partial y} \right) + \frac{\epsilon_t}{t^2} \left( -C_{\rho 1} f_{\rho 1} \left( ut \frac{\partial T}{\partial x} + vt \frac{\partial T}{\partial y} \right) - C_{d 1} f_{d 1} \epsilon_t \right) + \frac{\epsilon_t}{k} \left( -C_{\rho 2} f_{\rho 2} uv \frac{\partial U}{\partial y} - C_{d 2} f_{d 2} \epsilon \right) + \alpha \alpha_t \left( 1 - f_\lambda \right) \left( \frac{\partial^2 T}{\partial y^2} \right)^2$$

The upper and the under parts of the code appended at the buoyant components of each transport equation mean the aiding and the opposing flows, respectively. The following compound code means the same order. Note that Boussinesq approximation was introduced into the buoyant term of the momentum transport equation.

A Gradient-diffusion model using eddy viscosity and thermal eddy viscosity has been adopted to describe the turbulent fluxes that appear in the transport equations. However, there is a large discrepancy between the empirical data and the numerical data of the turbulent Prandtl number. Yin et al. [6] has therefore proposed the following forms of the turbulent fluxes of natural convection along a vertical flat plate to avoid the discrepancy,

$$\begin{aligned}
 -uv &= \nu_t \frac{\partial U}{\partial y} \pm C_b \tau_m g\beta \alpha_t \frac{\partial T}{\partial y} \\
 -vt &= \alpha_t \frac{\partial T}{\partial y} \\
 -ut &= -C_h \frac{1}{k} \nu_t \frac{\partial U}{\partial y} \alpha_t \frac{\partial T}{\partial y} \mp C_b \tau_m g\beta t^2
 \end{aligned}$$

Though these forms are the conventional gradient-diffusion model in theory, the buoyant components were added to the expressions of  $-uv$  and  $-ut$  as the second term in each right-hand side. In the calculation of the buoyant induced turbulence, we have only added a buoyant term to the turbulent transport equations until now, and there is very little research effort

of numerical computation in which the buoyant effect is accurately handled in the modeling for the turbulent fluxes. In the present study, the above expressions were examined on the numerical computation for the turbulent statistic of the combined convection that has not completely been argued. There is no necessity of assuming an experimentally-unknown turbulent Prandtl number when coupling the two-equation turbulence models for both the velocity and the temperature fields. The turbulent Prandtl number are derived from the following form,

$$Pr_t = \frac{v_t}{\alpha_t} = \frac{uv \frac{\partial T}{\partial y}}{vt \frac{\partial U}{\partial y}}$$

The model functions and constants are as follows,

#### Model functions and constants

$$\begin{aligned} \sigma_k &= 1.4, \sigma_\epsilon = 1.3, \sigma_h = \sigma_\phi = 1.0, C_{\epsilon 1} = 1.45, C_{\epsilon 2} = 1.9, \\ C_{d1} &= 2.2, C_{d2} = 0.8, C_\mu = 0.1, C_\lambda = 0.11, C_b = 0.7, C_h = 1.0, \\ C_{p1} &= 1.8, C_{p2} = 0.72, f_{p1} = f_{p2} = f_{d1} = f_{d2} = 1.0 \\ f_\mu &= [1 - \exp(-y_k^+ / 7.8)]^2, \quad f_\lambda = [1 - \exp(-y_k^+ / 9.0)]^2, \\ f_1 &= 1.0, f_2 = 1 - 0.3 \exp(-R_t^2), \quad R_t = k^2 / \nu / \epsilon, \\ v_t &= C_\mu f_\mu k \tau_m, \quad \alpha_t = C_\lambda f_\lambda k \tau_m, \quad \tau_m = \sqrt{(k / \epsilon)(t^2 / 2 / \epsilon_t)} \end{aligned}$$

The handling like the follow was applied to the damping function instead of using  $y^+ (=yu^*/\nu)$  determined from a friction velocity in order to avoid the inconvenience for the shear free phenomenon that may appear in the opposed flow of the turbulent combined convection,

$$y_k^+ = \frac{yu_k^*}{\nu} \quad \left( u_k^* = \sqrt{\nu \frac{\partial \sqrt{\kappa}}{\partial y} \Big|_w} \right)$$

As is shown in the form, the new damping function is composed of a gradient of square root of turbulent kinetic energy near wall. The handling has already been proposed for a shear free turbulence [7], where a local Kolmogoroff scale has also been introduced to the damping function. Abe et al. [8] have successively simulated the turbulent flow with separation and reattachment by applying the characteristic scale. The numerically-determined constants included in the damping function were modified to fit the computed data when  $Gr^* = 0$  to the empirical data of turbulent forced convection. The influence of changing the constants on the computation is therefore negligible.

The boundary conditions for the present numerical computation are as follows,

#### Boundary conditions

$$\begin{aligned} y = 0 : U = k = \epsilon = t^2 = \epsilon_t = 0, \quad q_w = \text{const} \\ y = H / 2 : \frac{\partial U}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial k}{\partial y} = \frac{\partial \epsilon}{\partial y} = \frac{\partial t^2}{\partial y} = \frac{\partial \epsilon_t}{\partial y} = 0 \end{aligned}$$

The flow is the fully-developed turbulent combined convection between vertical smooth parallel plates, where the inertia force acts on parallel or opposite to the buoyant force. The wall is heated uniformly.

#### **Numerical Procedure**

The calculation was carried out by TDMA method after differentiating the transport equation group with the aid of the control volume method [9] under a fixed flow rate condition. The lattice system is composed of lattice of total of 101 points as unequal interval lattice that becomes smaller near wall in the half-value width between the parallel plates. The initial condition was made to be a convergence value of the pure forced convection without the buoyancy. The convergence solution for each flow condition was obtained when the relative residual of all variables was less than  $1 \times 10^{-7}$  during all the computation.

#### **COMPUTATIONAL RESULTS AND DISCUSSION**

##### **Heat Transfer**

Figures A-1 & A-2 show the simulated heat transfer coefficients of the aiding and the opposing flows along with the previous conventional data of heat transfer. The non-dimensional parameter,  $Gr/(Re^3 Pr^{0.5})$ , derived from normalizing the momentum equation, stands for the effect of the buoyancy on the inertia force. The parameter is independent of all the length scales. The buoyancy becomes marked with shifting to the right side in the abscissa. The non-dimensional parameter,  $Nu/Nu_f$ , stands for the ratio of the combined convection heat transfer to the forced convection heat transfer without the buoyancy. The description "Model" means the computed results ( ) by using a conventional two-equation turbulence model when hypothesizing  $Pr_t = 0.9$ . Except for the numerical data involved in Figures A-1 & A-2, all the data are of various empirical investigations. Note that individual length scale is a hydraulic diameter. It is obvious that the present computed data ( ) coincide with the conventional data and that the present model is useful in simulating the heat transfer. The damping function proposed is also useful in simulating the heat transfer phenomenon. As is described in the previous chapter, there is an inconvenience for the shear free phenomenon that may appear in the opposed flow. Figure 1 indicates the inconvenience for the opposing flow when using a damping function without the modification. The computed result illustrated with does not indicate the good agreement with the empirical data of the heat transfer.

In the aiding flow, it is well-known that the heat transfer reduction occurs under some thermal hydraulics conditions [10, 11, 12, 13, 14, 15]. The heat transfer reduction often causes a problem that the performance of a high-temperature thermal equipment lowers in accordance with lowering of the material strength due to the rapid wall temperature rise. Clarifying the turbulent transport mechanism of the combined convection

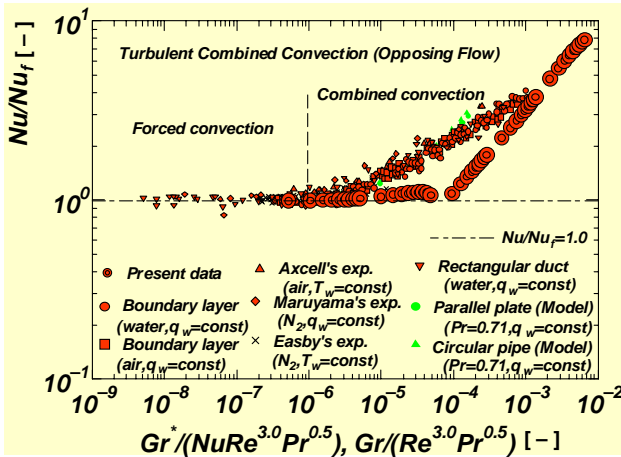


Figure 1 Heat Transfer Coefficients (Opposing Flow)

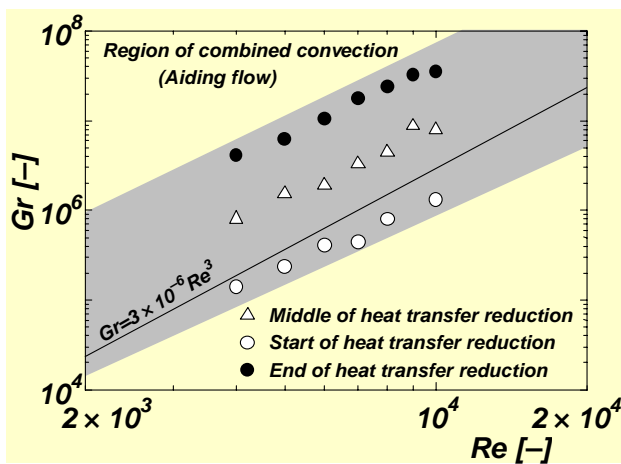


Figure 2 Region of Turbulent Combined Convection (Aiding Flow)

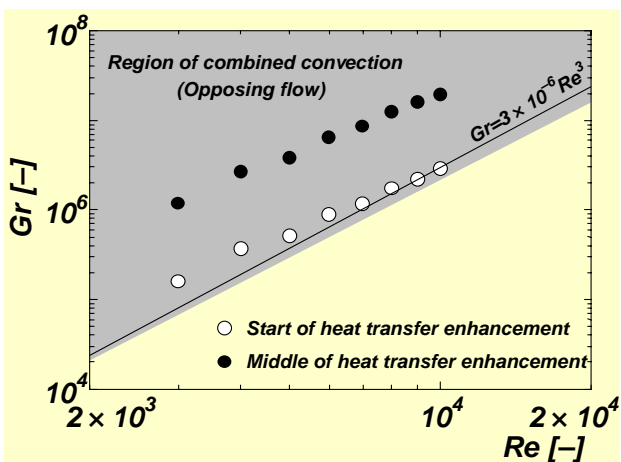


Figure 3 Region of Turbulent Combined Convection (Opposing Flow)

experimentally and numerically is therefore very important for the industrial utilizations when designing a large scale heat exchanger or handling a cooling accident of nuclear facilities, etc.. For example, it may develop to

a more serious situation, if the knowledge of the heat transfer reduction which is peculiar to the combined convection is not obtained in terms of the durability. In particular, the knowledge becomes important when the flow demand lowers owing to some accidents and causes in the maintenance and management of a cooling system that accompanies the nuclear facilities. The heat transfer quantity may do lower by running the coolant of a negligibly small flow rate than that of natural convection where the flow perfectly stops.

Figures 2 & 3 illustrate the combined convection region judged from the  $Re - Gr$  plane for the aiding and the opposing flows. It is obvious that the start of the heat transfer reduction & enhancement corresponds to the empirical correlation [16],  $Gr = 3 \times 10^6 Re^3$ , which stands for the onset of retransition from the turbulent flow to the laminar flow. The gray-colored parts mean the regions corresponding to the turbulent combined convection. Note that “Middle of heat transfer enhancement” in Figure 3 stands for the case when the enhancement ratio becomes 50 %.

### Transport Mechanism

Figures 4 & 5 present the simulated velocity distributions. It is obvious from Figure 4 that the form has gradually shifted from the forced convection mode to the natural convection mode with increasing  $Gr^*$  when  $Re$  is fixed. Figures 6 & 7 present the simulated temperature distributions. The temperature gradient near wall of the aiding flow once decreases for the case where the heat transfer reduction occurs. That of the opposing flow increases with increasing  $Gr^*$  in accordance with the heat transfer enhancement.

Figures 8 & 9 present the simulated kinetic energy distributions. The kinetic energy of the aiding flow once decreases for the case where the heat transfer reduction occurs. That of the opposing flow increases with increasing  $Gr^*$ . Figures 10 & 11 present the simulated Reynolds stress distributions. The Reynolds stress of the aiding flow once decreases for the case where the heat transfer reduction occurs. Then, the absolute amount increases with increasing  $Gr^*$  after changing the code. The form of the distribution has gradually shifted from the forced convection mode to the natural convection mode with increasing  $Gr^*$  when  $Re$  is fixed. The Reynolds stress of the opposing flow increases with increasing  $Gr^*$ . Figures 12 & 13 present the simulated distributions of turbulent heat flux. The turbulent heat flux of the aiding flow once decreases for the case where the heat transfer reduction occurs. In particular, the reduction near wall, where the turbulent production is generally activating in forced convection, becomes marked. The turbulent heat flux of the opposing flow increases with increasing  $Gr^*$ . It is recognized from a series of simulated turbulent behaviors that the heat transfer reduction is attributable to the turbulent suppression and that the heat transfer enhancement is attributable to the activation of turbulent production. Note that these figures do not accompany with the empirical

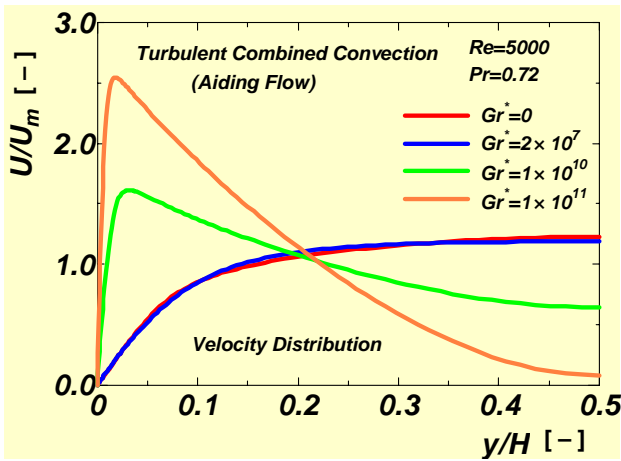


Figure 4 Velocity Distributions (Aiding Flow)

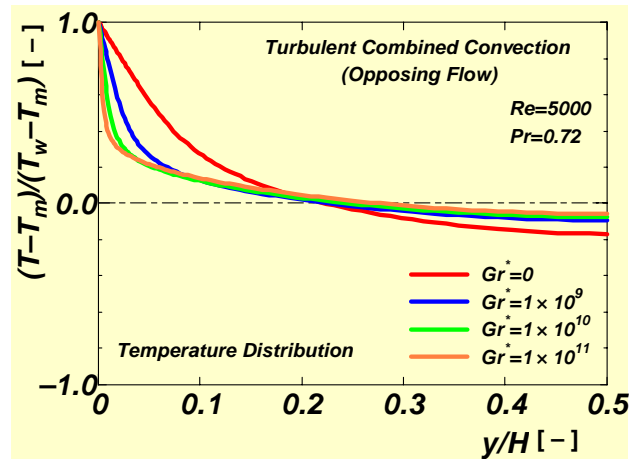


Figure 7 Temperature Distributions (Opposing Flow)

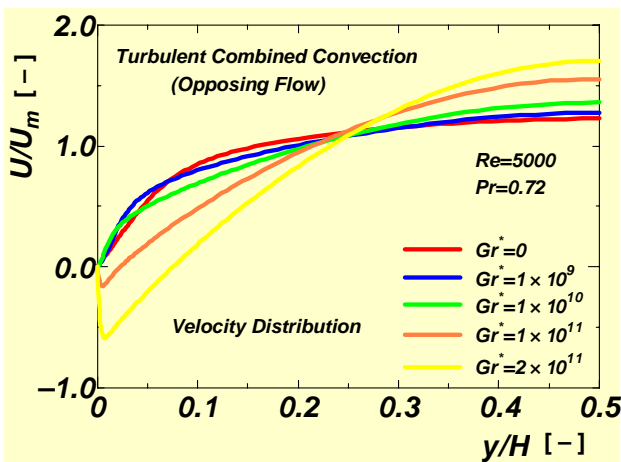


Figure 5 Velocity Distributions (Opposing Flow)

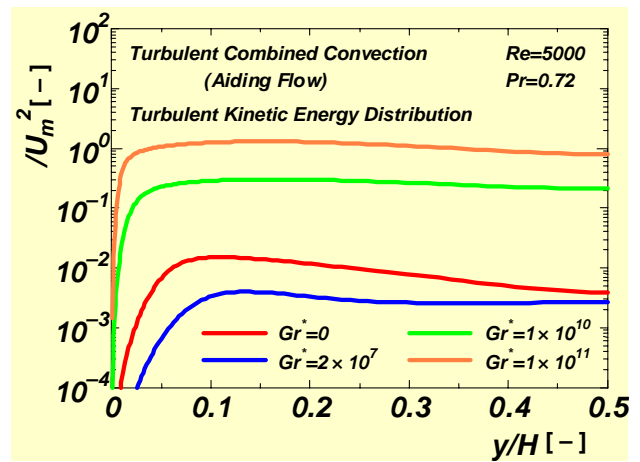


Figure 8 Kinetic Energy Distributions (Aiding Flow)

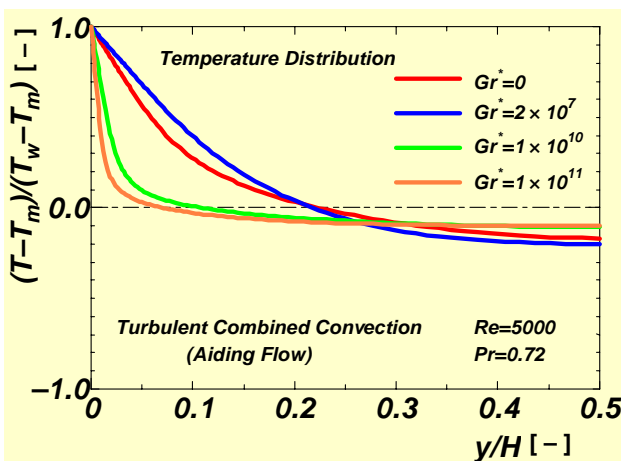


Figure 6 Temperature Distributions (Aiding Flow)

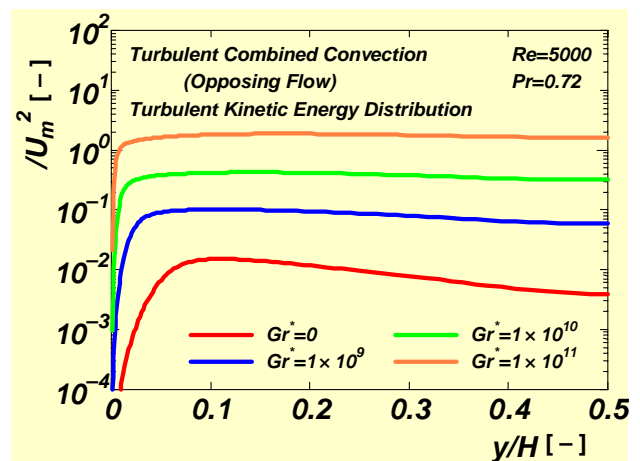


Figure 9 Kinetic Energy Distributions (Opposing Flow)

data, because there is no reliable experimental examinations on the turbulent statistic of the combined convection strongly affected by the buoyancy.

Figures 14 & 15 present the simulated distributions of turbulent Prandtl number. The turbulent Prandtl number of both the aiding and the opposing flows become infinite

near wall in the region of the turbulent combined convection strongly affected by the buoyancy. In the numerical computation by using conventional turbulence models based on the gradient-diffusion model, the Reynolds stress becomes zero at the maximum velocity point for the case where the buoyancy dominates in the

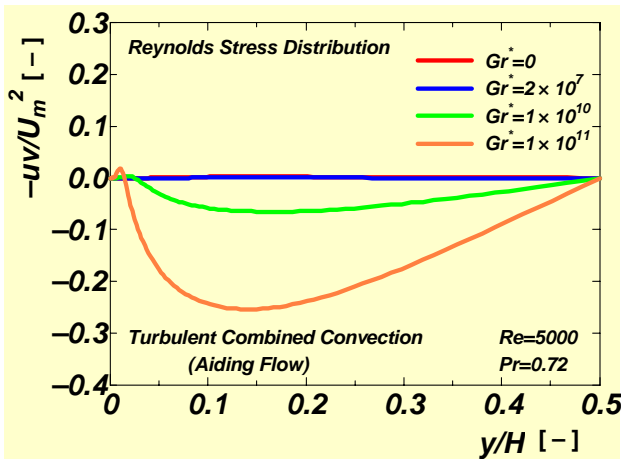


Figure 10 Reynolds Stress Distributions (Aiding Flow)

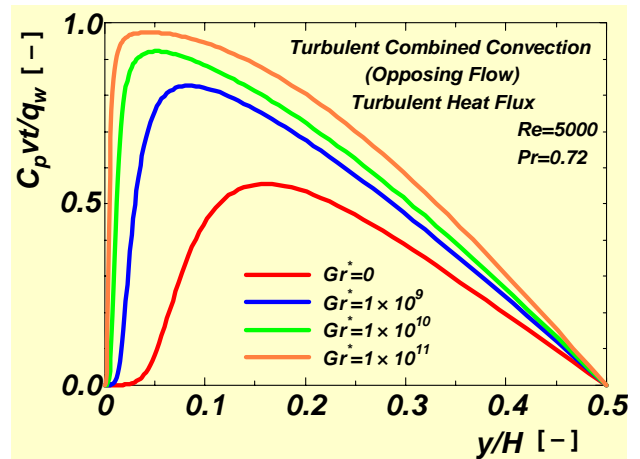


Figure 13 Distributions of Turbulent Heat Flux (Opposing Flow)

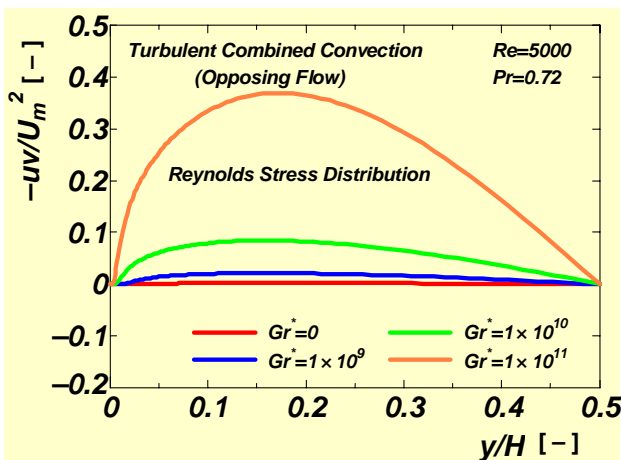


Figure 11 Reynolds Stress Distributions (Opposing Flow)

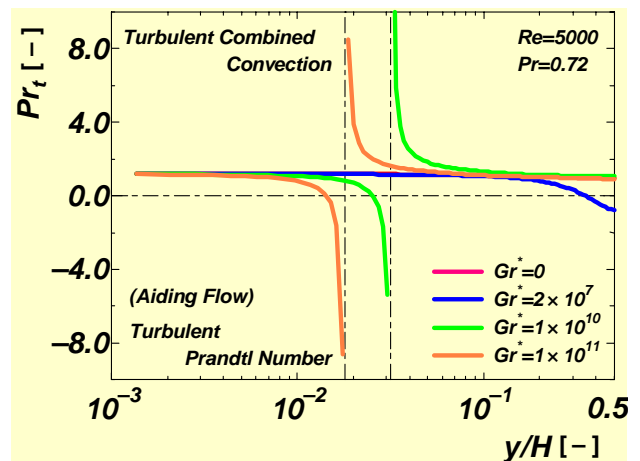


Figure 14 Distributions of Turbulent Prandtl Number (Aiding Flow)

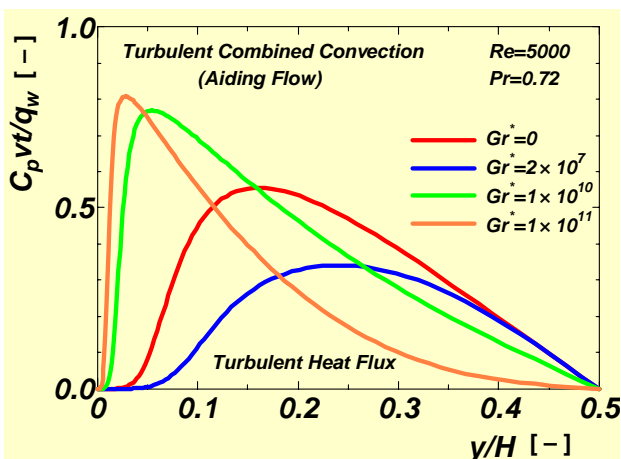


Figure 12 Distributions of Turbulent Heat Flux (Aiding Flow)

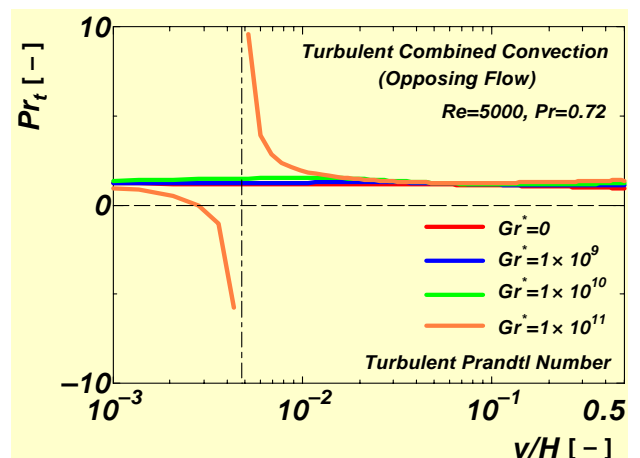


Figure 15 Distributions of Turbulent Prandtl Number (Opposing Flow)

flow field. The conventional models are not therefore applicable to reproduce the discontinuity of the turbulent Prandtl number. Moreover, there is the empirical fact that the turbulent Prandtl number distribution becomes discontinuous near wall in the turbulent natural

convection boundary layer [17, 18, 19]. Therefore, as well as natural convection, it is no wonder that there is a discontinuity in the distribution of the turbulent Prandtl

number of the combined convection, in which the maximum velocity exists. This fact of the discontinuity originates from the buoyant components added to the expressions of the turbulent fluxes; the buoyant components play a significant role in reproducing the behavior of the turbulent Prandtl number more realistically, and it is the matter that could not be guessed from various conventional turbulence models and various empirical data on the turbulent combined convection. Note that the reason why the turbulent Prandtl number at  $Gr^* = 2 \times 10^7$  becomes negative near the center in spite of the positive mean velocity gradient is the existence of the negative Reynolds stress as shown in Figure 10.

### CONCLUDING REMARKS

The turbulent transport of the fully developed combined convection between vertical smooth parallel plates, where the inertia force acts on parallel and opposite to the buoyant force, was investigated numerically by combining the two-equation turbulence models of the different type being applicable to the velocity and the temperature fields. This work is a first trial to simulate the turbulent combined convection by applying the modeling of the turbulent fluxes and by improving the damping function. It was obvious that the modified model including the proposed damping function is appropriate to simulate the heat transfer and fluid flow of the turbulent combined convection physically.

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